

# Are All Boer-Mulders Functions Alike?

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Chirally odd generalized parton distributions (GPDs) and the Boer-Mulders function provide valuable information about spin-orbit correlations for quarks in nucleons and other hadrons. We compare results for the relevant GPD  $\bar{E}_T^q$  from a variety of phenomenological models as well as recent lattice results. We find that  $\bar{E}_T^q > 0$  for nucleons as well as the pion and for both  $u$  and  $d$  quark. As a result, the corresponding Boer Mulders functions are all expected to be negative. The sign of  $\bar{E}_T^q$  arises from the relative sign between the upper and lower Dirac components for the quark wave functions.

## I. INTRODUCTION

Recent experiments by the HERMES collaboration have demonstrated the existence of significant single-spin asymmetries (SSAs) in semi-inclusive deep-inelastic scattering (SIDIS) [1]. The Sivers function [2] measured in these experiments indicates a negative correlation between the transverse proton polarization and the transverse momentum for ejected  $u$  quarks, while the same correlation for  $d$  quarks turned out to be positive (Note that we adhere to the Trento convention [3]).

Meanwhile, significant advances have been made in the theoretical understanding of SSAs [4]. For example, simple model calculations have illustrated very clearly that SSAs do not need to vanish in the Bjorken limit [5]. However, it has also become clear that several ingredients need to conspire in order to produce SSAs [6]: essentially what is needed is an interference between final state interaction phases for quarks originating from different partial waves in the target wave function. Clearly, such a complex interplay between different aspects of hadron structure not only makes SSAs interesting, but it also presents a challenge for theorists to produce predictions that do not depend sensitively on model parameters. Indeed, due to the complexity of these calculations, it even took a while until agreement was reached in observables as simple as the sign of the Sivers function in the scalar diquark model.

Significant clarification regarding the sign issue was accomplished once it was noticed that the sign of these SSAs can be intuitively related to the sign of the deformations of certain spin-dependent impact parameter dependent parton distributions [7, 8]: the final state interactions (FSIs) are expected to be attractive, on average, and therefore impact parameter dependant parton distributions (IPDs), which are purely intrinsic properties of hadrons, can be related to SSAs, which also involve FSIs. In the case of the Sivers effect, the relevant IPD is the Fourier transform of the Generalized Parton Distribution (GPD)  $E^q(x, 0, -\Delta_\perp^2)$  that describes the  $x$ -decomposition of the Pauli form factor  $F_2^q(-\Delta_\perp^2)$ . Since the latter is approximately known for  $q = u, d$ , it was possible to predict the sign of the  $u$  and  $d$  quark Sivers functions [9, 10, 11, 12] which were subsequently confirmed by the HERMES data [1].

Even when the target nucleon is unpolarized, the momentum distribution of its quarks ejected in SIDIS can already exhibit a left-right asymmetry of the transverse quark momentum  $\mathbf{k}_T$  relative to their own transverse spin  $\mathbf{S}_q$  [13]

$$f_{q^\uparrow/p}(x, \mathbf{k}_T) = \frac{1}{2} \left[ f_1^q(x, k_T^2) - h_1^{\perp q}(x, k_T^2) \frac{(\hat{\mathbf{P}} \times \mathbf{k}_T) \cdot \mathbf{S}_q}{M} \right]. \quad (1)$$

$\hat{\mathbf{P}}$  is a unit vector opposite to the direction of the virtual photon momentum and  $M$  is the target mass. This asymmetry can be measured experimentally by tagging the transverse polarization of quarks produced in SIDIS using the Collins effect [14]. Alternatively, one can also study the  $\cos 2\phi$  asymmetry in the unpolarized Drell-Yan process, where  $\phi$  is the azimuthal angle of the  $\mu^+\mu^-$  plane about the virtual photon axis w.r.t. the incident proton. Like the Sivers function, the “Boer-Mulders function”  $h_1^{\perp q}(x, k_T^2)$  requires a combination of orbital angular momentum with a difference between the final state interaction from different parts of the initial quark orbit. Since the Boer-Mulders function specifies the correlation between the transverse momentum asymmetry of the struck quark and its spin, it provides important information about the correlation between quark orbital angular momentum and spin. Similar to the case of the Sivers function, one expects that the sign of  $h_1^{\perp q}$  can be related to the distribution of transversely polarized quarks in impact parameter space [15], which can be obtained from the Fourier transform of chirally odd

GPDs

$$\int \frac{dz^-}{4\pi} e^{ixP^+z^-} \langle p' | \bar{q}(-\frac{1}{2}z) \sigma^{+j} \gamma_5 q(\frac{1}{2}z) | p \rangle = \frac{1}{2P^+} \left[ H_T(x, \xi, t) \bar{u} \sigma^{+j} \gamma_5 u + \tilde{H}_T(x, \xi, t) \bar{u} \frac{\epsilon^{+j\alpha\beta} \Delta_\alpha P_\beta}{M^2} u \right. \\ \left. + E_T(x, \xi, t) \bar{u} \frac{\epsilon^{+j\alpha\beta} \Delta_\alpha \gamma_\beta}{2M} u + \tilde{E}_T(x, \xi, t) \bar{u} \frac{\epsilon^{+j\alpha\beta} P_\alpha \gamma_\beta}{M} u \right] \quad (2)$$

$$= \frac{1}{2P^+} \left[ \left( H_T - \frac{t}{2M^2} \tilde{H}_T \right) \bar{u} \sigma^{+j} \gamma_5 u + \tilde{H}_T \bar{u} \frac{\Delta^j \sigma^{+\alpha} \gamma_5 \Delta_\alpha - \Delta \sigma^{j\alpha} \gamma_5 \Delta_\alpha}{2m^2} u \right. \\ \left. + (E_T + 2\tilde{H}_T) \bar{u} \frac{\epsilon^{+j\alpha\beta} \Delta_\alpha \gamma_\beta}{2M} u + \tilde{E}_T(x, \xi, t) \bar{u} \frac{\epsilon^{+j\alpha\beta} P_\alpha \gamma_\beta}{M} u \right]. \quad (3)$$

The linear combination  $\bar{E}_T = E_T + 2\tilde{H}_T$  multiplying the last term in (3) describes the transverse deformation of the distribution of transversely polarized quarks in an unpolarized nucleon [16]. In ref [15] it has been suggested that  $h_1^{\perp q}$  and  $\bar{E}_T^q$  have opposite signs. Therefore, knowledge of  $\bar{E}_T^q$  make qualitative predictions for the Boer-Mulders function possible. However, not even the sign of  $\bar{E}_T^q$  is known experimentally. The main purpose of this paper is to develop some intuition about the expected sign for  $\bar{E}_T$  by analyzing this observable in a variety of hadron models.

## II. IMPACT PARAMETER DEPENDENT PDFS IN THE BAG MODEL

The well known MIT bag model offers a familiar venue to begin our discussion of the sign of SSAs in the nucleon. Bag model wavefunctions have the form

$$\Psi_m = \frac{N}{\sqrt{4\pi}} \begin{pmatrix} ij_0 \chi_m \\ -j_1(\vec{\sigma} \cdot \hat{x}) \chi_m \end{pmatrix}, \quad (4)$$

where  $\chi_m$  is a Pauli spinor, and the  $j_n$  are the spherical Bessel functions and  $N$  is a normalization factor. In order to slightly broaden the scope of our discussion, note that  $j_1$  related to  $j_0$  by a derivative. In fact, in the bag model, this relation between the upper and lower component is a consequence of the (free) Dirac equation. With this in mind, let us instead focus on a more general Dirac spinor,

$$\Psi_m = \begin{pmatrix} if \chi_m \\ -g(\vec{\sigma} \cdot \hat{x}) \chi_m \end{pmatrix}, \quad (5)$$

where  $f$  is a monotonically decreasing radial function, and  $g$  is the derivative of  $f$ .

The impact parameter dependent parton distributions that we would like to evaluate are of the form

$$F_\Gamma(x, \mathbf{b}_\perp) = \mathcal{N}^{-1} \int \frac{dz^-}{4\pi} e^{ixp^+z^-} \langle p^+, \mathbf{0}_\perp | \bar{q}(0, \mathbf{b}_\perp) \Gamma q(z^-, \mathbf{b}_\perp) | p^+, \mathbf{0}_\perp \rangle. \quad (6)$$

Here  $|p^+, \mathbf{0}_\perp\rangle$  is an eigenstate of light-cone momentum  $p^+$  as well as of the transverse center of momentum operator  $\mathbf{R}_\perp$  (for a definition of the latter see Refs. [7, 17]).

From the bag model wave functions one can easily evaluate position space densities of quark bilinears, but evaluating parton distributions requires introducing additional recipes in order to deal with light-like correlation functions. This issue can be entirely avoided by focusing on the lowest moment of GPDs, yielding a (local) density that only depends on  $\mathbf{r}_\perp$ .

$$\int dx F_\Gamma(x, \mathbf{b}_\perp) = (2p^+ \mathcal{N})^{-1} \langle p^+, \mathbf{0}_\perp | \bar{q}(0, \mathbf{b}_\perp) \Gamma q(0, \mathbf{b}_\perp) | p^+, \mathbf{0}_\perp \rangle. \quad (7)$$

Instead of evaluating the matrix element of the operator in Eq. (7) at the origin between plane wave states, one may equivalently localize also the longitudinal position of the state and instead integrate the operator over longitudinal position, yielding

$$\int dx F_\Gamma(x, \mathbf{b}_\perp) = \text{const.} \int dx^3 \langle \tilde{\mathbf{0}} | \bar{q}(x^3, \mathbf{b}_\perp) \Gamma q(x^3, \mathbf{b}_\perp) | \tilde{\mathbf{0}} \rangle, \quad (8)$$

where  $|\tilde{\mathbf{0}}\rangle$  denotes a nucleon state localized at the origin (in all three space directions). Note that, as the bag model is not boost invariant, there is a certain arbitrariness in the extraction of  $\bar{E}_T$  from bag model wave functions. In

particular, the reference point for the impact parameter  $\mathbf{b}_\perp$ , which should be the transverse center of longitudinal momentum, is here taken to be simply the center of the spherical bag. However, since we are only concerned about the overall sign of  $\bar{E}_T$ , such issues should not be significant.

Quarks with transverse polarization  $\mathbf{s}$  are projected out by the operator  $\frac{1}{2}\bar{q}[\gamma^+ - s^j i\sigma^{+j}\gamma_5]q$  [16] and therefore the vector field representing the transverse quark polarization density is given by  $-i\bar{q}\sigma^{+j}\gamma_5 q$ . We thus consider impact parameter dependent PDFs with  $\Gamma = -i\sigma^{+j}\gamma_5$  which are related to the (Fourier transforms of the) chirally odd GPDs  $\bar{E}_T$ ,  $H_T$  and  $\tilde{H}_T$  [16]

$$F_T^i = -\epsilon^{ij}b^j \frac{1}{M}\bar{\mathcal{E}}'_T + S^i \left( \mathcal{H}_T - \frac{1}{4M^2}\Delta_b \tilde{\mathcal{H}}_T \right) + (2b^i b^j - b^2 \delta^{ij}) S^j \frac{1}{M^2} \tilde{\mathcal{H}}''_T \quad (9)$$

where we used script letters to denote the Fourier transforms of these GPDs and  $S^j$  is the spin of the target. Only the term involving  $\bar{\mathcal{E}}_T$  contributes for an unpolarized target (averaging over transverse target polarization), which is why it is only the GPD  $\bar{E}_T$  that is expected to be (qualitatively) related to the BM function. In the bag model, we extract this term by considering the density corresponding to  $\Gamma = -i\sigma^{+j}\gamma_5$  and summing over the target spin. For a single quark state this procedure yields

$$\sum_m \langle PS_m | \bar{\Psi}(x^3, \mathbf{b}_\perp) i\sigma^{+i}\gamma_5 \Psi(x^3, \mathbf{b}_\perp) | PS_m \rangle = -\frac{1}{\sqrt{2}} \sum_m (f^2 + g^2) s_m^i + 2fg\epsilon^{ij}\hat{b}_\perp^j - 2g^2\hat{b}_\perp^i (\hat{b}_\perp \cdot \vec{s}_m) \quad (10)$$

where  $\vec{s}_m$  is the spin vector corresponding to the pauli spinor  $\chi_m$ . The first and last terms of (10) do not survive the sum over ‘target’ polarizations. The asymmetry is given entirely by the middle term, which is an interference between the upper and lower components of (5). For the lowest moment of  $\bar{\mathcal{E}}_T$ , we find

$$\kappa_T = \int dx \bar{E}_T(x, 0, 0) = \int dx d^2\mathbf{b}_\perp \bar{\mathcal{E}}_T = \frac{2MG}{3\sqrt{2}\pi} \int_0^{R_0} dr r^3 fg. \quad (11)$$

The right hand side of (11) is always positive because  $f$  and  $g$  are non-negative functions for  $r$  less than the bag radius, implying that  $\bar{\mathcal{E}}_T \geq 0$ .

In the bag model, the correlation between quark spin and quark orbital motion is the same, regardless of the orientation of  $j_z$ . All quark spin orientations thus contribute coherently to  $\bar{\mathcal{E}}_T^q$  and in the case of  $d$  quarks,  $\bar{\mathcal{E}}_T^d$  is equal to  $\bar{\mathcal{E}}_T^q$  for a single quark, while for  $u$  quarks it is twice as large. In fact, for any model where the quarks are confined by some mean field potential one finds that all quark orbits give the same contribution to  $\bar{\mathcal{E}}_T^q$  and thus  $\bar{\mathcal{E}}_T^q$  is equal to  $\bar{\mathcal{E}}_T^q$  for a single quark orbit, multiplied by the number of quarks of flavor  $q$ . In particular, in the large  $N_C$  limit, where  $N_u = N_d + 1 \rightarrow \infty$ , the lowest  $x$  moment of  $\bar{\mathcal{E}}_T^q$  is the same for  $u$  and  $d$  quark and both are of order  $\mathcal{O}(N_C)$ . Since the support of GPDs shrinks to  $x = \mathcal{O}(1/N_C)$ , this implies that  $\bar{E}_T^u(x, \xi, t) = \bar{E}_T^d(x, \xi, t) = \mathcal{O}(N_C^2)$ .

In order to visualise the transverse spin - impact parameter correlation in the bag model, the vector field

$$-\int dx^3 fg\epsilon^{ij}\hat{b}^j \quad (12)$$

representing the lowest moment of the transversity density in an unpolarized target has been plotted in Fig. 1 for bag model wave functions  $f = j_0(r)$ , and  $g = j_1(r)$ . In the bag model, we thus obtain a counter-clockwise polarization for impact parameter dependent quark distributions. Upon invoking the ‘‘chromodynamic lensing’’ mechanism (attractive FSI), and using the Trento convention [3], this implies a negative Boer-Mulders function  $h_1^\perp$ . This result is also consistent with a direct calculation of  $h_1^\perp$  for the bag model in Ref. [18]. The most important question here is: how general is this result regarding the sign of  $h_1^\perp$ , i.e. how much does it hinge on specific features of the bag model.

In order to address this issue, let us consider the general case of a particle confined by a combination of scalar potential (i.e. a mass term  $m(r)$  that depends on the radius) as well as a (zero component of a) vector potential  $V(r)$ . The bag model corresponds to the limiting case where the vector potential vanishes and the scalar potential has the shape of an infinite square well. For a free quark, the lower component  $\phi_l$  is related to the upper component  $\phi_u$ , via the free Dirac equation

$$\phi_l = \frac{1}{E + m} \vec{\sigma} \cdot \vec{p} \phi_u. \quad (13)$$

In the presence of an external potential this relation changes only slightly

$$\phi_l = \frac{1}{E + m(r) - V(r)} \vec{\sigma} \cdot \vec{p} \phi_u. \quad (14)$$

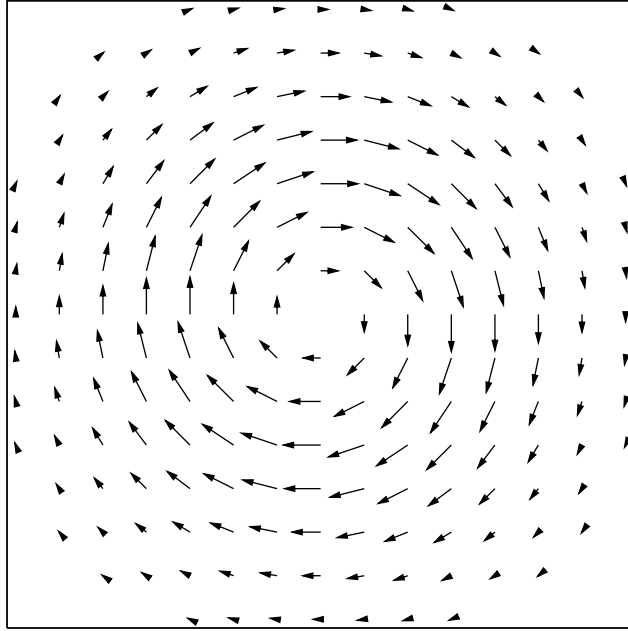


FIG. 1: Lowest moment of the impact parameter dependent transversity distribution for an unpolarized target in the MIT bag model (12). The ‘outside’ of the spherical bag corresponds to the regions without arrows.

The vector part of the confinement potential cannot exceed the scalar part — otherwise one encounters the Klein paradox. Therefore,  $E + m - V$  should in general be positive. As a result the sign in this relation is unchanged by the presence of the potentials  $m(r)$  and  $V(r)$ . In particular, since one would expect that the upper component  $f(r)$  for the ground state is a monotonically falling positive function (its overall phase has been taken positive here), one expects  $g(r)$  to be negative, regardless of the details of the potentials. Note that while  $E + m - V$  can be negative at isolated points without giving rise to the Klein paradox, e.g. very close to the origin for an electron moving in a Coulomb potential, this does not invalidate our main point regarding the average value of the product  $f(r) \cdot g(r)$  or  $f(r) \cdot g(r)r$  relevant for the lowest moments of  $\bar{E}_T$  at zero momentum transfer.

This discussion illustrates that the sign of the spin-orbit correlation (counterclockwise) described by Eq. (10) should be the same for the ground state of all confining potential models.

### III. DIQUARK MODELS

One of the main disadvantages of the bag model is its lack of boost invariance, which results in certain ambiguities in the parton interpretation of bag model wave functions. In diquark models the distribution of quarks in a ‘nucleon’ is obtained by allowing it to split into a quark and a spectator ‘diquark’. As the wave function is usually calculated perturbatively using a point-like vertex, boost invariance and electromagnetic gauge invariance are straightforward to maintain. The same applies to (perturbatively calculated) QCD final state interactions between the quark and the diquark. Because of the latter features, it is thus not surprising that the scalar diquark model has provided the first clear and convincing example for the fact that SSAs can survive in the Bjorken limit [5].

The phenomenological motivation for diquark models is the idea that, as long as the momentum transfer on the spectators is not very large, one may replace them by a single, point-like degree of freedom — the diquark. The quark-quark interaction in the nucleon is more attractive in the scalar channel with quark spins anti-aligned. This further motivates the assumption that the spectator diquark carries scalar quantum numbers. In this work, we will mostly focus on the scalar diquark model only, although more sophisticated diquark models exist.

The scalar diquark model wave function, has only two spin components corresponding to positive ( $\uparrow$ ) and negative quark helicity ( $\downarrow$ ). Angular momentum conservation implies that the wave function components for ‘nucleon’ states with positive ( $\uparrow$ ) helicity take on the form

$$\begin{aligned}\psi_{\uparrow}^{\uparrow} &= f(x, \mathbf{k}_{\perp}^2) \\ \psi_{\downarrow}^{\uparrow} &= (k^x + ik^y)g(x, \mathbf{k}_{\perp}^2)\end{aligned}\tag{15}$$

Time reversal implies for the negative helicity states ( $\Downarrow$ )

$$\begin{aligned}\psi_{\uparrow}^{\Downarrow}(x, \mathbf{k}_{\perp}) &= (-k^x + ik^y)g(x, \mathbf{k}_{\perp}^2) \\ \psi_{\downarrow}^{\Downarrow}(x, \mathbf{k}_{\perp}) &= f(x, \mathbf{k}_{\perp}^2).\end{aligned}\tag{16}$$

For transversely polarized quarks (lower index) in a transversely polarized target (upper index) this implies

$$\psi_{+\hat{x}}^{+\hat{x}}(x, \mathbf{k}_{\perp}) = f(x, \mathbf{k}_{\perp}^2) + ik^y g(x, \mathbf{k}_{\perp}^2)\tag{17}$$

$$\psi_{-\hat{x}}^{+\hat{x}}(x, \mathbf{k}_{\perp}) = -k^x g(x, \mathbf{k}_{\perp}^2),\tag{18}$$

i.e. in impact parameter space

$$\begin{aligned}\tilde{\psi}_{+\hat{x}}^{+\hat{x}}(x, \mathbf{c}_{\perp}) &\equiv \int d^2\mathbf{k}_{\perp} e^{i\mathbf{k}_{\perp} \cdot \mathbf{c}_{\perp}} \psi_{+\hat{x}}^{+\hat{x}}(x, \mathbf{k}_{\perp}) = \tilde{f}(x, \mathbf{c}_{\perp}^2) + \frac{\partial}{\partial c^y} \tilde{g}(x, \mathbf{c}_{\perp}^2) \\ \tilde{\psi}_{-\hat{x}}^{+\hat{x}}(x, \mathbf{c}_{\perp}) &\equiv \int d^2\mathbf{k}_{\perp} e^{i\mathbf{k}_{\perp} \cdot \mathbf{c}_{\perp}} \psi_{-\hat{x}}^{+\hat{x}}(x, \mathbf{k}_{\perp}) = i \frac{\partial}{\partial c^x} \tilde{g}(x, \mathbf{c}_{\perp}^2)\end{aligned}\tag{19}$$

As a result of the interference between the helicity non-flip  $\tilde{f}$  and helicity-flip component  $\partial_y \tilde{g}$ , the distribution of quarks with the same transverse polarization as the ‘nucleon’  $\left| \tilde{\psi}_{\hat{x}}^{+\hat{x}}(x, \mathbf{c}_{\perp}) \right|^2$  exhibits a transverse asymmetry proportional to  $\tilde{f} \partial_y \tilde{g}$ . The wave function component where the transverse quark spin is opposite to the nucleon spin does not exhibit such an asymmetry. Intuitively, one can understand the origin of the asymmetry in the scalar diquark model as follows. When the ‘nucleon’ splits into a quark and a diquark, the orbital angular momentum of the two particle system can be either  $l = 0$  or  $l = 1$  (Note that this is not in conflict with parity invariance as upper and lower components of quark spinors transform differently under parity and leading twist quark densities result from superpositions of upper and lower components). Higher orbital angular momenta are not possible as the total angular momentum (OAM plus spin of the quark) must still be  $\frac{1}{2}$ . For quarks that have the same (transverse) spin as the ‘nucleon’, the orbital wave function can have either  $l = 0$  or  $l = 1$  and  $l_x = 0$ . The asymmetry results from the interference between these two wave function components very much in the same way as hybridization in chemistry results in asymmetric electron orbitals. In contradistinction, the wave function component where the quark spin is opposite to the ‘nucleon’ spin is pure  $l = 1$ , i.e. there cannot be any interference between  $l = 0$  and  $l = 1$  and hence no asymmetry. This very simple observation also ‘explains’ why the BM and Sivers functions are identical in the scalar diquark model (for an explicit calculation of both functions in this model see Ref. [19]): a left-right asymmetry only occurs in the wave function component where the transverse spin of the quark and the transverse spin of the ‘nucleon’ are the same. Thus the correlation between the transverse position space asymmetry with the quark spin is the same that it is with the nucleon spin. Since (at least in diquark models) the transverse momentum space asymmetry in SIDIS arises entirely from the initial transverse position space asymmetry, the BM function (representing the correlation between transverse momentum and spin of the struck quark) and the Sivers function (representing the correlation between transverse quark momentum and ‘nucleon’ spin) must be the same. While it has been known that those two T-odd functions are the same in the scalar diquark model, the intuitive explanation provided here is new.

For models containing axial vector diquarks the situation is more complicated, as there can also be interference between wave function components where the transverse quark spin is opposite to the ‘nucleon’ spin (and the vector diquark spin is along the ‘nucleon’ spin direction). As a result, the correlation between quark spin and its transverse position is no longer identical to the correlation between ‘nucleon’ spin and quark transverse position. Hence the Sivers and BM functions in vector diquark models do not have to be identical.

In the scalar diquark model

$$\begin{aligned}\psi_{+\hat{y}}^{+\hat{y}}(x, \mathbf{c}_{\perp}) &= \left[ M + \frac{m}{x} + \frac{1}{x} \frac{d}{dc^y} \right] \phi(\mathbf{c}_{\perp}^2) \\ \psi_{-\hat{y}}^{+\hat{y}}(x, \mathbf{c}_{\perp}) &= -\frac{i}{x} \frac{d}{dc^y} \phi(\mathbf{c}_{\perp}^2)\end{aligned}\tag{20}$$

where  $\mathbf{c}_{\perp}$  is the distance between the active quark and the spectator and  $\phi(\mathbf{c}_{\perp})$  is monotonically decreasing with  $\mathbf{c}_{\perp}^2$ . This result is of the above form (19), with  $\tilde{f} = \left[ M + \frac{m}{x} \right] \phi$  and  $\tilde{g} = \frac{1}{x} \phi$ , i.e. quarks polarized in the  $+\hat{x}$  direction are shifted towards negative  $y$ . Again, we find a counter-clockwise pattern for the transverse quark polarization corresponding to a positive GPD  $\tilde{E}_T^q$ . A qualitatively similar result is obtained for axial vector diquark models.

It is easy to understand why we obtain the same sign as in the bag model. The quark interacts with the diquark (to form the nucleon) only when the two are at the same point, i.e. in impact parameter space the interaction is a contact

interaction. For nonzero impact parameter, the quark is thus a free particle and as such subject to the free Dirac equation relating upper and lower component. Just as was the case in the bag model, this determines the relevant phase which in turn determines the sign of  $\bar{E}_T^q$ . This observation has far reaching consequences for other (e.g. axial vector) diquark models. What all diquark models have in common is the fact that quark and diquark interact only when they merge to become the nucleon, i.e. the quark-diquark interaction is local. Both for scalar and axial vector diquarks, the the quark-diquark state is mainly an  $s$  state and therefore the same general statements regarding the sign of  $\bar{E}_T^q$  apply to both the axial vector and scalar case.

#### IV. RELATIVISTIC CONSTITUENT QUARK MODELS

In relativistic constituent quark models for hadron structure, one starts from nonrelativistic forms for the quark wave function. Nontrivial spin structure is then generated by boosting these wave functions to the infinite momentum frame. In this procedure, using non-interacting boost operator, the net result is that instant form spin eigenstates  $|k^3, \mathbf{k}_\perp; \pm\rangle_I$  are replaced by light-front helicity eigenstates  $|k^+, \mathbf{k}_\perp; \pm\rangle_F$  using what is often referred to as a Melosh transformation [20]

$$\begin{aligned} |k^3, \mathbf{k}_\perp; +\rangle_I &= w [(k^+ + m) |k^+, \mathbf{k}_\perp; +\rangle_F - k^R |k^+, \mathbf{k}_\perp; -\rangle_F] \\ |k^3, \mathbf{k}_\perp; -\rangle_I &= w [(k^+ + m) |k^+, \mathbf{k}_\perp; -\rangle_F + k^L |k^+, \mathbf{k}_\perp; +\rangle_F] \end{aligned} \quad (21)$$

where  $w = 1/\sqrt{2k^+(m+k^0)}$ ,  $k^{R/L} = k^1 \pm ik^2$ , and  $k^+ = k^0 + k^3$ . For quarks with nonzero transverse momentum, the boost to the infinite momentum frame (or equivalently the Melosh transformation) naturally introduces a correlation between quark spin and quark orbital angular momentum and a non-trivial spin structure of the nucleon state — even if the original (instant form) wave function only contained s-wave components and SU(6) wave functions.

Such models have been used to estimate  $\bar{E}_T^q$  in Ref. [21]. Studying a wide range of model parameters, it was found that  $\kappa_\perp^u$  in the range between 1.98 and 3.60, and  $\kappa_\perp^d$  in the range between 1.17 and 2.36. The sign of these results is in agreement with the results from the other models. Furthermore, similar to the bag model, one finds  $\kappa_u \approx 2\kappa_d$ . It is possible to understand the origin of the sign for  $\bar{E}_T$  in this model. The relativistic constituent models start from pure  $s$  wave wavefunctions. Any  $p$  wave component is generated by the Melosh transformation — which employs the free (noninteracting) boost operator. It is thus not surprising that the phase relation between the  $s$  and  $p$  wave is thus the same as for all the other models studied here.

#### V. $\bar{E}_T$ IN THE PION

Since the pion has no spin, its impact parameter dependent PDFs are much simpler than in the case of the nucleon. The distribution of quarks with spin  $s^i$  in impact parameter space reads

$$\frac{1}{2} [F + s^i F_T^i] = H(x, \mathbf{b}^2) + s^i \epsilon^{ij} b^j \frac{2}{m} \frac{\partial}{\partial \mathbf{b}^2} \bar{E}_T(x, \mathbf{b}^2), \quad (22)$$

where  $\mathcal{H}(x, \mathbf{b}^2)$  and  $\bar{\mathcal{E}}_T(x, \mathbf{b}^2)$  are again the fourier transforms of the GPDs  $H(x, 0, t)$  and  $\bar{E}_T(x, 0, t)$  respectively whose definition is particularly simple

$$\int \frac{dz^-}{4\pi} e^{ixP^+z^-} \langle \pi' | \bar{q}(-\frac{1}{2}z) \gamma^+ q(\frac{1}{2}z) | \pi \rangle |_{z^+=0, z=0} = H(x, \xi, t) \quad (23)$$

$$\int \frac{dz^-}{4\pi} e^{ixP^+z^-} \langle \pi' | \bar{q}(-\frac{1}{2}z) \sigma^{+j} \gamma_5 q(\frac{1}{2}z) | \pi \rangle |_{z^+=0, z=0} = \frac{1}{\Lambda} \bar{E}_T(x, \xi, t) \frac{\epsilon^{+j\alpha\beta} \Delta_\alpha P_\beta}{P_+}. \quad (24)$$

Here  $\Lambda$  is some hadronic mass scale, which needs to be included in the definition if one wants  $\bar{E}_T(x, \xi, t)$  to be dimensionless (cf. Eq. 3). For the nucleon, the natural choice is the nucleon mass, but choosing  $\Lambda = m_\pi$  for the pion would perhaps not be a wise choice, as this would unnecessarily complicate the discussion of the chiral limit. Since we are again mainly interested in the sign of  $\bar{E}_T$ , we leave this issue open, but perhaps  $\Lambda = 4\pi f_\pi$  or  $\Lambda = M_N$  would be more useful choices for the pion case.

Except for a slight change in the bag radius, the quark wave functions in the bag model are the same for pions and nucleons. Therefore, apart from a slight rescaling due to the different bag radii,  $\bar{E}_T^u$  in a  $\pi^+$  is the same as  $\frac{1}{2}\bar{E}_T^u$  or  $\bar{E}_T^d$  in the proton. Here the factor  $\frac{1}{2}$  accounts for the fact that there are twice as many  $u$  quarks in a proton than in a  $\pi^+$ .

As an alternative model for GPDs in the pion we also considered the Nambu-Jona-Lasino (NJL) model. In this model, the quark and the anti-quark interact via a  $\gamma_5$  contact interaction in the  $s$ -channel, and form factors or GPDs are obtained by evaluating momentum integrals containing quark traces of the form

$$\frac{\text{tr} [\gamma_5(k + \frac{\Delta}{2} + M)\Gamma(k - \frac{\Delta}{2} + M)\gamma_5(k - p + M)]}{[(k + \frac{\Delta}{2})^2 - M^2][(k - p)^2 - M^2][(k - \frac{\Delta}{2})^2 - M^2]}. \quad (25)$$

Here  $M$  is the (constituent) quark mass and  $\Gamma$  depends on the current under consideration. For example,  $\Gamma = \gamma^+$  is used to calculate PDFs and the GPD  $H$ , yielding for the trace

$$\text{tr}[\Gamma = \gamma^+] = 4 \{p^+(M^2 - k^2) + k^+[M^2 - (p - k)^2 + p^2]\}. \quad (26)$$

Evaluating the expression when the spectator is on-shell  $[(p - k)^2 = M^2]$ , which arises from complex countour intagrations for  $k^-$ , and dropping all terms proportional to  $m_\pi^2$  this yields

$$\text{tr}[\Gamma = \gamma^+] = 4p^+ \frac{M^2 + \mathbf{k}_\perp^2}{p^+ - k^+} = 4 \frac{M^2 + \mathbf{k}_\perp^2}{1 - x}. \quad (27)$$

Together with a factor  $\frac{1}{1-x}$  in the energy denominators, this gives rise to the well known result of a constant PDF in the NJL model (for  $m_\pi = 0$ ).

In the calculation of  $\Delta_i \bar{E}_T$ , it is  $\Gamma = \gamma^+ \gamma^i$  which enters the above trace. For simplicity, we focus here on  $\bar{E}_T$  for  $\Delta \rightarrow 0$  which determines the average spin dipole moment  $\kappa_\perp$ . Since the denominator is even in  $\Delta$ , only the  $\Delta$ -dependence in the numerator matters in this limit, and we thus consider only the relevant numerator

$$\begin{aligned} \text{tr}[\Gamma = \gamma^+ \gamma^i] &= \text{tr} \left[ (k + \frac{\Delta}{2} + M) \gamma^+ \gamma^i (k - \frac{\Delta}{2} + M) (p - k + M) \right] = \frac{\Delta_i}{2} \text{tr} [(k + M) \gamma^+ + \gamma^+ (k + M)] (p - k + M) \\ &= 4 \Delta_i M p^+, \end{aligned} \quad (28)$$

yielding again  $\bar{E}_T > 0$ . A more detailed analysis yields  $\bar{E}_T \propto (1-x)$ , for  $m_\pi \rightarrow 0$ , where the constant of proportionality depends on the cutoff in the loop integrals.

Enlightened by our discussion about the sign of  $\bar{E}_T$  in the nucleon, it is easy to understand the result in the NJL model: the interaction between quark and antiquark in this model is also a contact interaction, i.e. the quark and antiquark follow the free Dirac equation, except when they sit on top of each other. Therefore, by the same reasoning as in the bag model,  $\bar{E}_T$  should be positive, as the explicit calculation confirmed. Lattice calculations yield the same sign [22].

## VI. SUMMARY AND CONCLUSIONS

We have studied the chirally odd GPD  $\bar{E}_T$  for both  $u$  and  $d$  quarks in a variety of models (Bag model, potential models, diquark model, NJL-model, relativistic constituent quark models). All models provide the same sign  $\bar{E}_T > 0$ . This sign also agrees with recent results from lattice QCD calculations. While the physical origin of the sign is obscure in lattice QCD calculations, it is much more transparent in the model calculations. As the quark helicity flips in the matrix elements contributing to  $\bar{E}_T$ , the nucleon helicity does not. Hence the quark orbital angular momentum between the initial and final state must differ by one unit and therefore  $\bar{E}_T$  arises from the interference between wave function components that differ by one unit of orbital angular momentum. In many models for the structure of ground state hadrons only  $s$  and  $p$  wave components are present and higher OAM is either completely absent or negligible. Moreover, in these models, the  $p$ -wave component arises primarily from the ‘lower’ component of the hadron wave function, which is related to the upper component via the Dirac equation, which thus determines the relative phase between the upper and lower components. While the presence of interactions tends to modify the relation between the upper and lower components quantitatively, it does not change the phase relation and all models yield the same phase relation between upper and lower component that also holds for free quarks. Lattice results confirm this result. Of course, one needs to be cautious as present day lattice calculations are still performed at moderately high quark masses. The pion masses used in Ref. [23] range from  $m_\pi^2 \approx 1 \text{ GeV}^2$  to  $m_\pi^2 \approx 0.2 \text{ GeV}^2$ . and the fact that the phase relation between upper and lower components is the same as for the free Dirac equation could in principle be an artifact of the large quark mass. However, the results from Ref. [23] are also rather stable against variations of the quark mass, and the sign of  $\bar{E}_T^q$  does not change upon extrapolation to the physical quark masses, yielding  $\kappa_T^u \approx 2.93$  and  $\kappa_T^d \approx 1.90$ .

Such a unanimous agreement between such a variety of theoretical approaches to an observable that has never been measured is probably almost unprecedented. While some of the model predictions for the sign of  $\bar{E}_T^q$  existed before, what is new in this work is the intuitive understanding about the sign of  $\bar{E}_T^q$  as resulting from the relation between upper and lower components in the (free) Dirac equation. Despite all the complexities of QCD, lattice calculations yield results that are qualitatively consistent with quarks in an  $s$  state that have a  $p$ -wave in the lower (Dirac) component of the wave function and with a relative phase obtained from the free Dirac equation. While this agreement may be a pure accident, it is still very tempting to interpret this result as an indication that a nontrivial fraction of the orbital angular momentum in the nucleon wave function is merely resulting from the  $p$ -wave admixture in the lower component of quark spinors in a bound state.

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